

# Quantitative Analysis of High Temperature Protective Clothing Design Based On Solution and Optimization Model of Fractional Partial Differential Equations

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**Abstract:** This paper is based on the calculation of thermal conduction fractional partial differential equations, combined with function interpolation and nonlinear optimization. The measurement software programming calculation and actual results are finally obtained under the use of fractional-order partial differential equations and nonlinear optimization algorithms to obtain the optimal thickness design of high-temperature protective clothing under the multi-layer heat transfer temperature distribution law and the actual limited working conditions.

## 1. Introduction

The high-temperature protective clothing is a protective clothing that can prevent the body from being uncomfortable in a high-temperature environment. Excellent thermal protective clothing not only has a good barrier effect on external heat, but also achieves a certain heat and moisture transfer capacity to facilitate heat release of body and sweat evaporation. However, in thermal protective clothing, it is always contradictory to enhance thermal protective performance and reduce human metabolic heat load. Therefore, the design of high temperature thermal protective clothing is of great significance for the efficient completion of high temperature operation and related scientific research. In this paper, we establish a mathematical model to study the changes in the temperature of the material layer in the high temperature thermal protective clothing when the temperature outside the surface of the dummy skin is constantly changing. Under the limitation of the actual high temperature working conditions, the material for high temperature thermal protective clothing is known.

## 2. Assumption of data

The data in this paper is mainly from the 2018 National College Students Mathematical Modeling Competition. In order to solve the parameters more conveniently, the following hypotheses are proposed. Firstly, the uniformity of the same fabric layer in different directions in the high-temperature protective clothing and the absorption of heat in different areas of the same fabric layer are the same. Secondly, the volume change of the high temperature protective clothing caused by changes in moisture and humidity is negligible. Thirdly, the heat transfer process in the "environment-apparel-skin" system only considers the heat transfer process of heat conduction and heat radiation. Fourthly, the fabric layers in the high temperature protective clothing do not decompose during the heat prevention process. Lastly, the radiation coefficient of different fabric layers in the high temperature protective clothing is fixed.

### 3. Study on temperature distribution of material layer based on thermal conduction fractional partial differential equation

#### 3.1 Research ideas

Special clothing usually consists of three layers of fabric material, which are labeled as I, II and III layers. The I layer is in contact with the external environment, and there is still a gap between the III layer and the skin, which is labeled as IV layer.

According to the different levels of density, specific heat, thermal conductivity and thickness, the heat transfer efficiency of the three layers of fabric can be reflected. However, it is necessary to use the known parameter information for an ambient temperature of 75 ° C and a thickness of II layer. The temperature distribution of the III layer can be solved under the condition of 6 mm, IV layer thickness of 5 mm and working time of 90 minutes. The parameters of heat transfer efficiency, ambient temperature, fabric thickness and working time of different fabrics can be studied. The fabric level (I, II, III) is regarded as a system with the same heat conduction method but different parameters.

#### 3.2 Research methodology

In the temperature field, select any point of the same at a certain moment, that is  $y=f(x, y, z, r)$  on the first-class temperature line, then describe the maximum temperature change rate of any point with  $\text{grad}t$  which is short for temperature gradient, namely  $\text{grad}t = \frac{\partial t}{\partial n}n$ . The heat flux conducted per

unit area perpendicular to the density is expressed by the thermal conductivity  $\lambda = \frac{q}{-\text{grad}t}$  to

indicate the thermal conductivity of the uniform medium. On this basis, the Fourier theorem for

thermal conduction satisfaction is shown as  $dQ = -k \frac{\partial u}{\partial n} ds dt = -k \nabla u ds dt$ . It indicates that the heat  $dQ$

flowing along the normal direction of the unit area  $ds$  is proportional to the temperature change rate

$\frac{\partial u}{\partial n}$  on both sides of the area element in  $dt$  time, where the proportional coefficient  $k$  is the thermal

conductivity described above.

Combined with the law of energy conservation and the ratio of heat and density, the Fourier

theorem can be rewritten to  $\frac{\partial u}{\partial t} = \frac{k}{cp} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$ . It is also worth noting that, in the initial

stage, the surface temperature of the false skin also plays a thermal conduction role in the fourth

layer of protective clothing. Also in the  $dt$  time, the heat generated at any point inside the object is  $Q$

$(x, y, z, t)$ , resulting in a thermal conduction equation containing a heat source as  $u^2 = a^2 \Delta u + f, f = \frac{F}{cp}$ .

##### (1) Fractional Partial Differential Equation Model in High Temperature Protective Clothing

Firstly, consider the heat transfer in the high-temperature protective clothing, and establish the equations according to the temperature  $T(\alpha, t)$  of the high-temperature protective clothing and the heat radiation from the left and right boundaries of the high-temperature protective clothing.

$$\begin{cases} C_1^A(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial a} \left( K_1(T) \frac{\partial T}{\partial a} \right) + \frac{\partial H_1(a, t)}{\partial a} + \frac{\partial H_2(a, t)}{\partial a} (a, t) \in A_1 \times (0, t_1) \\ C_2^A(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial a} \left( K_2(T) \frac{\partial T}{\partial a} \right) (a, t) \in A_2 \times (0, t_1) \\ C_3^A(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial a} \left( K_3(T) \frac{\partial T}{\partial a} \right) (a, t) \in A_3 \times (0, t_1) \end{cases}$$

The contact surface between the first layer and the second layer in the high temperature protective

$$\text{clothing is calculated as } \begin{cases} T|_{\alpha=\alpha_1} = T|_{\alpha=\alpha_1} \\ -K_2 \frac{\partial T}{\partial \alpha}|_{\alpha=\alpha_1} = -K_1 \frac{\partial T}{\partial \alpha}|_{\alpha=\alpha_1} \\ (1-\beta_2)H_1(\alpha_1, t) + \beta_1 \delta T^4(\alpha_1, t) = H_2(\alpha_1, t) \end{cases}$$

Secondly, the initial condition of the void layer of the IV-layer fractional partial differential equation model is  $T(\alpha, 0) = T_1(\alpha)$ . Then, the fractional partial differential equation model of the void layer is  $C_4^A(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial \alpha} \left( K_4(T) \frac{\partial T}{\partial \alpha} \right) - \frac{\partial p_3}{\partial \alpha}(\alpha, t) \in A_4 \times (0, t_1)$

### 3.3 Question solving

Numerical dispersion of fractional differential equations established for the “environment-apparel-skin” system.

$0 = \dot{n}_1 < \dot{n}_2 < \dots < \dot{n}_{a_1} = a_1$  Indicates the division of the first layer of fabric layer;  
 $a_1 = \ddot{n}_1 < \ddot{n}_2 < \dots < \ddot{n}_{a_2} = a_2$ , indicates the division of the second layer of fabric layer;  
 $a_2 = \bar{n}_1 < \bar{n}_2 < \dots < \bar{n}_{a_3} = a_3$ , indicates the division of the third layer of fabric layer;  
 $a_3 = \hat{n}_1 < \hat{n}_2 < \dots < \hat{n}_{a_4} = a_4$ , indicates the division of the fourth layer of fabric layer.

So, the dispersion of the first three fabric layers is  $C^A(T^{n+1}) \frac{\dot{T}_i^{n+1} - \ddot{T}_i^n}{T_1} = \frac{K(T^{n+1}) - K(T_i^{n+1})}{h_1} \times \frac{T_i^{n+1} - T_i^{n+1}}{h_1} + K(T_{i+1}^{n+1}) \frac{T_i^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}}{h_1} + \Theta_i^{n+1}$

If  $i = 1, 2, \dots, a_1 - 1; n = 0, 1, \dots, N - 1$ , and  $\omega = \frac{\tau_0}{ph_1^2}, v = \frac{\tau_0}{p}$ , then

$$- \omega \frac{K(T_i^n)}{c(T_i^n)} T_{i-1}^{n+1} + [1 + \omega \left( \frac{K(T_i^n) + K(T_{i+1}^n)}{c(T_i^n)} \right)] T_i^{n+1} - \omega \frac{K(T_{i+1}^n)}{c(T_i^n)} T_{i+1}^{n+1} = T_i^n + v \Theta_i^n$$

## 4. Research on optimal thickness of single layer material layer based on nonlinear optimization algorithm

### 4.1 Particle clustering algorithm for the use of optimal thickness design

#### 4.1.1 Research methods

Assume that the high temperature protective clothing I, II, III layer is gathered into the outer layer, the fourth layer air gap is the inner layer, and the inner and outer thickness of the fabric are respectively  $A1', A2'$ , then the total thickness  $A' = A1' + A2'$ .

Based on the heat transfer model established above, the heat conduction process of layer II can be obtained. The results are as follows  $q^n = \delta \varepsilon (T_i^4 - T_j^4) - \delta \varepsilon F (1 - \varepsilon) (T_i^4 - T_j^4)$

For  $\alpha_1 = 0.06$  mm, it can be determined that the thickness of layer II  $\alpha_2 = (A1 - 0.6)$  mm,  $\alpha_3 = 3.6$  mm,  $\alpha_4 = 5.5$  mm. And the initial conditions for layer II are  $T(\alpha, 0) = T(\alpha_2, 0) = T(A1 - 0.6, 0)$ .  $T(x, t = 3600) \leq 47$  °C  $T(x, t \leq 3600) \leq 44$  °C. Combine the above equations and use the Lingo operation to get  $\alpha_2 \geq 9.2$ . On the other hand, the particle swarm method is used to numerically solve the positive heat transfer question. Select the threshold  $\varepsilon$  and the maximum number of iterations  $N_{max}$ . The initial position and initial velocity of each particle are as  $\vec{Z}_i^{(0)} = (z_{i1}, \dots, z_{id}, \dots, z_{iD})$ ,

$$\vec{V}_i^{(0)} = (v_{i1}, \dots, v_{id}, \dots, v_{iD}), i = 1, 2, \dots, M$$

### 4.1.2 Analysis Conclusion

It can be seen from the output that, when the ambient temperature is 65 °C, the thickness of the IV layer is determined and equal to 5.5 mm, at  $t \leq 3600$  s, the constraint condition  $T(x, t = 3600) \leq 47$  °C,  $T(x, t \leq 3600) \leq 44$  °C is satisfied, from the non-steady state to the steady state and the production cost, the optimal thickness of the layer II may be 9.2 mm.

## 4.2 Simple Genetic Algorithm for Analysis of Optimal Thickness of Single Layer Material Layer

### 4.2.1 Research ideas

For the optimum thickness of the II and IV layers when the ambient temperature is 80 °C, ensure that the outside temperature of the dummy's skin does not exceed 47 °C and the time of more than 44 °C does not exceed 5 minutes when working for 30 minutes. And the question of the degree of thermal damage to the skin was transformed into the question of the degree of thermal damage to the skin, and a theoretical model of the opposite direction was established.

### 4.2.2 Research process

In order to improve the thickness of protective clothing for high temperature operation, Henriques integral is used as a standard. When the temperature of the base layer reaches 44 °C, the human skin will be thermally damaged. In addition, the personal safety of high temperature workers is the main consideration. In order to predict the degree of skin damage, the contact surface temperature is substituted into the following  $\Omega = \int_0^t P e^{\left(-\frac{\Delta E}{RT(x, \tau)}\right)} d\tau$

Among them, the time interval of integration is the time after the skin temperature outside the human body exceeds 44 °C.  $T$  takes the temperature of the substrate layer, and when the substrate layer is satisfied, thermal damage does not occur. Taking the skin contact surface on the outside of the human body as the target point, if  $\Omega \leq 1$  is satisfied, it is considered that no thermal damage occurs, and the critical  $\Omega$  where thermal damage occurs is recorded as  $\Omega_c$ . In addition, other parameters of the genetic algorithm are assigned. The number of individuals is 40, and the number of iterations is 100, also the number of variables is 2, then the 25-digit number of each variable is expressed as 25, as well as the number of generation grooves is 0.9. Here we will run the maximum of 50 times as the optimal solution, and we get  $A = (10.6, 4.2)$  and  $Q(A) = 0.9906$ . That is, the optimum thickness of layer II is 10.6 mm, and the optimum thickness of layer IV is 4.2 mm.

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